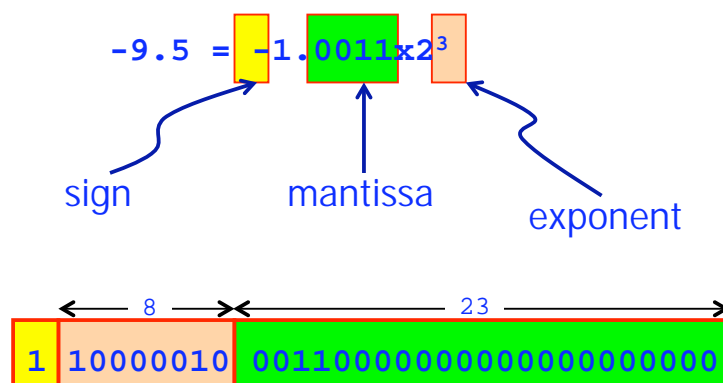


A Floating-Point Multiplier

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An overview of the IEEE FP format



- The number, in binary, must be normalized: the integer part must always be equal to 1
- The exponent, an integer value, is not represented in 2-complement, but in a biased representation: a bias of 127 is added to the exponent

- As the value 0 can not be normalized, a special representation is reserved for: all bits to zero
- In general, the values 00000000 and 11111111 from the exponent field are reserved for special cases and are not biased values:
 - 00000000 is used for non-normalized values
 - 11111111 is used for infinity and NaN (not a number)

Multiplication algorithm

- A multiplication of two floating-point numbers is done in four steps:
 - non-signed multiplication of mantissas: it must take account of the integer part, implicit in normalization. The number of bits of the result is twice the size of the operands (48 bits)
 - normalization of the result: the exponent can be modified accordingly
 - addition of the exponents, taking into account the bias
 - calculation of the sign

Example

- Let's suppose a multiplication of 2 floating-point numbers A and B, where A=-18.0 and B=9.5
- Binary representation of the operands:
A = -10010.0
B = +1001.1
- Normalized representation of the operands:
A = -1.001x2⁴
B = +1.0011x2³
- IEEE representation of the operands:
A = 1 10000011 001000000000000000000000
B = 0 10000010 001100000000000000000000

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- Multiplication of the mantissas:
 - we must extract the mantissas, adding an 1 as most significant bit, for normalization
1001000000000000000000000000
1001100000000000000000000000
 - the 48-bit result of the multiplication is:
0x558000000000
 - only the most significant bits are useful: after normalization (elimination of the most significant 1), we get the 23-bit mantissa of the result. This normalization can lead to a correction of the result's exponent
 - in our case, we get:

01 010101100000000000000000 000000000000000000000000

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- Addition of the exponents:

- exponent of the result is equal to the sum of the operands exponents. A 1 can be added if needed by the normalization of the mantissas multiplication (this is not the case in our example)
- as the exponent fields (Ea and Eb) are biased, the bias must be removed in order to do the addition. And then, we must to add again the bias, to get the value to be entered into the exponent field of the result (Er):

$$\begin{aligned} E_r &= (E_a - 127) + (E_b - 127) + 127 \\ &= E_a + E_b - 127 \end{aligned}$$

- in our example, we have:

Ea	10000011	
Eb	10000010	
-127	10000001	
Er	10000110	

what is actually 7, the exponent of the result

- Calculation of the sign of the result:

- the sign of the result (Sr) is given by the exclusive-or of the operands signs (Sa and Sb):

$$S_r = S_a \oplus S_b$$

- in our example, we get:

$$S_r = 1 \oplus 0 = 1$$

i.e. a negative sign

- Composition of the result:

the setting of the 3 intermediate results (sign, exponent and mantissa) gives us the final result of our multiplication:

1	10000110	010101100000000000000000
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$$A \times B = -18.0 \times 9.5 = -1.0101011 \times 2^{134-127} = -10101011.0 = -171.0_{10}$$

Laboratory

- Design, writing in VHDL, a hardware multiplier of 2 floating-point numbers A and B, represented using the simple precision IEEE format (32 bits)
- Synthesize your program for the Altera board and verify its behavior. Try for example with:
 - A=134.0625 and B=-2.25
 - A=-14.5 and B=-0.375
 - A=7.5 and B=15.5

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Solutions

- $A = 134.0625 = 1.00001100001 \times 2^7$
 $A = 0\ 10000110\ 000011000010000000000000 = 0x43061000$
 $B = -2.25 = -1.001 \times 2^1$
 $B = 1\ 10000000\ 001000000000000000000000 = 0xC0100000$
 $A \times B = -301.640625 = -1.00101101101001 \times 2^8$
 $A \times B = 1\ 10000111\ 001011011010010000000000 = 0xC396D200$
- $A = -14.5 = -1.1101 \times 2^3$
 $A = 1\ 10000010\ 110100000000000000000000 = 0xC1680000$
 $B = -0.375 = -1.1 \times 2^{-2}$
 $B = 1\ 01111101\ 100000000000000000000000 = 0xBEC00000$
 $A \times B = 5.4375 = 1.010111 \times 2^2$
 $A \times B = 0\ 10000001\ 010111000000000000000000 = 0x40AE0000$

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- $A = 7.5 = 1.111 \times 2^2$
 $A = 0\ 10000001\ 111000000000000000000000 = 0x40F00000$
 $B = 15.5 = 1.1111 \times 2^3$
 $B = 0\ 10000010\ 111100000000000000000000 = 0x41780000$
 $A \times B = 116.25 = 1.11010001 \times 2^6$
 $A \times B = 0\ 10000101\ 110100010000000000000000 = 0x42E88000$

